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Technical Comments

Comment on "Reply by Author to A. H. Flax"

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CHOPRA,[1] in his Reply to my Comment[2] on his Note,[3] seems to have missed one of the main points of my remarks. He states that, for the special cases of supersonic panel flutter which he treated, his procedures given in Ref. 1 will lead to the same results as those indicated in Ref. 2, the two procedures being merely alternative ways of arriving at the same answer. This is not the case. In fact, Chopra's new interpretation[1] of the formulas he presented in Ref. 3 leads to a recipe which cannot be carried out at all.

The transformation given in Ref. 2 relating the curve of λ vs g_T for a panel unrestrained by an elastic foundation to the corresponding curve for a panel restrained by an elastic foundation of stiffness, $K=K_I$, is unique and one to one for corresponding points on the curves. The transformation depends on a knowledge not only of g_T, the damping coefficient, and λ, the flutter speed parameter, but also of ω_F, the flutter frequency. With this information, it is possible to relate points on the two curves at the same value of λ by the formula[2]

$$g_{Te}=g_T/(1+K_I/m\omega_F^2)^{1/2} \tag{1}$$

where the subscript e refers to the panel on an elastic foundation. This transformation carries point A on the flutter boundary of the unrestrained panel to the point A' of the flutter boundary of the elastically restrained panel, as shown in Fig. 1.

If point A is, in fact, the flutter point of the unrestrained panel (corresponding to a given value of g_T) and if the only change in the physical parameters of the panel is the addition of the restraint of an elastic foundation, the flutter point of the restrained panel is the point B' on the transformed curve. However, there is no way to proceed directly from point A to point B'. Instead, the transformation to B' must be from point B. One cannot use Eq. (1) or any formula given by Chopra[3] to determine the value of damping corresponding to

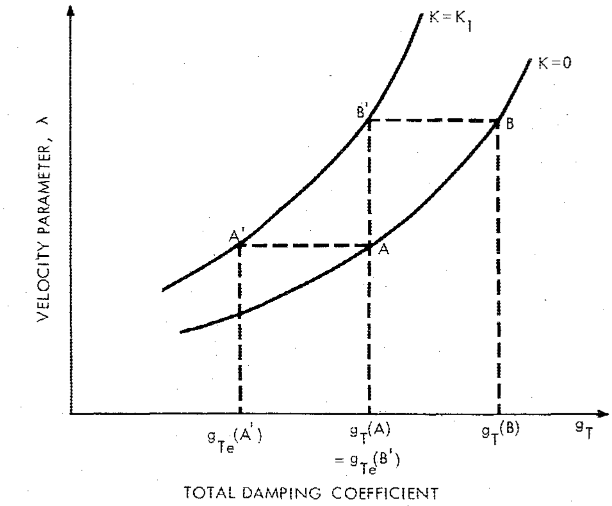

Fig. 1 Transformation of stability boundary.

point B, since the flutter frequency for point B is not known in advance.

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Comment on "Natural Frequencies of a Cantilever with Slender Tip Mass"

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IN Ref. 1, Bhat and Kulkarni have compared a perturbation solution attributed to Bhat and Wagner with the exact solutions for the vibration frequencies of a uniform cantilever beam with a tip mass M having an angular moment of inertia about its center of gravity J_0 for which the center of gravity may be displaced from the point of attachment of the beam by a distance L. Forming nondimensional quantities in terms of the length of the cantilever ℓ and its mass per unit length m,

$$\alpha=M/m\ell, \quad \epsilon=L/\ell, \quad \beta=J_0/m\ell^3 \tag{1}$$

are defined. The unperturbed problem has given α and β with $\epsilon=0$; ϵ is the perturbation parameter.

The authors of Ref. 1 conclude that the perturbation method, including terms up to the second order, gives good results except for the case $\beta=0$. Unfortunately, their formulas for the perturbation method appear to be in error, and this, in turn, gives rise to large errors in their numerical results for the perturbation method in the case $\beta=0$, leading finally to their incorrect conclusion that the perturbation method fails in some way for $\beta=0$. The first-order errors in the numerical

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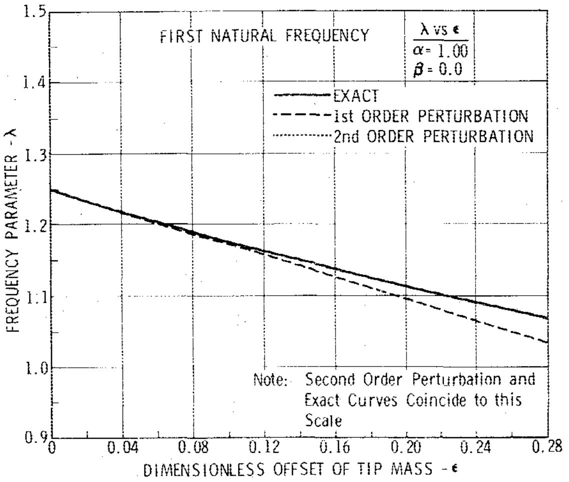

Fig. 1 Variation of first natural frequency with tip mass location.

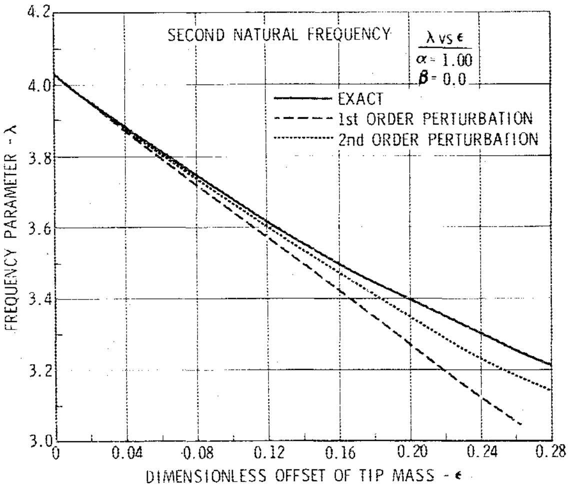

Fig. 2 Variation of second natural frequency with tip mass location.

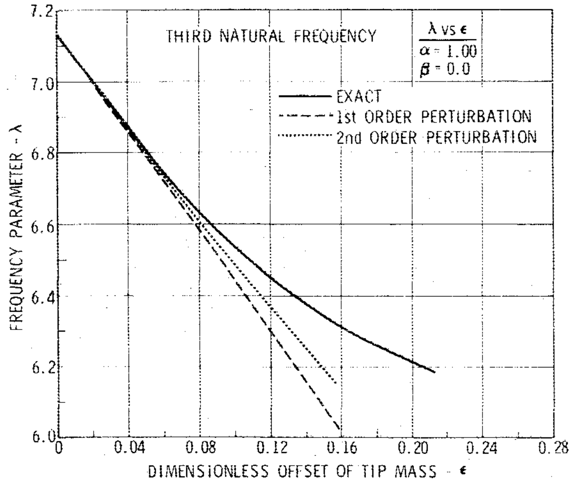

Fig. 3 Variation of third natural frequency with tip mass location.

results of Ref. 1 are obvious by inspection of the authors' Fig. 2, since the curves of frequency parameter against perturbation parameter derived from the perturbation method are not tangent to the curves representing the exact solution at the point where the perturbation parameter is zero. This condition of tangency is required, since mathematically the perturbation solution is equivalent to a Taylor's series expansion of the exact solution for frequency about the point where the perturbation ϵ is zero.

The usual perturbation expansion for the beam frequency parameter λ_n is

$$\lambda_n = \lambda_{n0} + \epsilon\lambda_{n1} + \epsilon^2\lambda_{n2} + \ldots \tag{2}$$

λ is given by

$$\lambda^4 = m\omega^2\ell^4/EI$$

where ω is the vibration frequency and EI is the bending stiffness of the uniform beam.

The characteristic frequency equation and the solution for vibration modes for this problem can be derived in closed form for both the unperturbed and the perturbed problems considered in Ref. 1. In such cases, resort to perturbation methods is unnecessary and may be an unattractive alternative to calculating the exact solution in view of the fact that the accuracy of the results of the perturbation analysis may be uncertain. The special circumstances, however, make it possible to carry out the perturbation analysis directly and simply by differentiation of the exact characteristic equation for frequency rather than having to use the more general and somewhat more laborious analysis of Rayleigh's classical perturbation theory.[2] For the case $\beta = 0$, the characteristic frequency equation reduces to

$$1 + G(\lambda) + \alpha\lambda[H(\lambda) - J(\lambda)] - 2\alpha\epsilon\lambda^2 F(\lambda) - \alpha\epsilon^2\lambda^3[H(\lambda) + J(\lambda)] = 0 \tag{3}$$

where

$$F(\lambda) = \sinh\lambda\sin\lambda; \quad G(\lambda) = \cosh\lambda\cos\lambda \tag{4a}$$

$$H(\lambda) = \sinh\lambda\cos\lambda; \quad J(\lambda) = \cosh\lambda\sin\lambda \tag{4b}$$

The series expansion coefficients for λ in Eq. (2) are obtained by differentiation of Eq. (3) as

$$\lambda_{n1} = \frac{2\alpha\lambda_{n0}^2 F(\lambda_{n0})}{(1+\alpha)[H(\lambda_{n0}) - J(\lambda_{n0})] - 2\alpha\lambda_{n0}F(\lambda_{n0})} \tag{5}$$

and

$$\lambda_{n2} = \{\alpha(\lambda_{n0}\lambda_{n1}^2 + 2\lambda_{n0}^2\lambda_{n1} + \lambda_{n0}^3)[H(\lambda_{n0}) + J(\lambda_{n0})] + (4\alpha\lambda_{n0}\lambda_{n1} + \lambda_{n1}^2 + 2\alpha\lambda_{n1}^2)F(\lambda_{n0})\}/\{(1+\alpha)[H(\lambda_{n0}) - J(\lambda_{n0})] - 2\alpha\lambda_{n0}F(\lambda_{n0})\} \tag{6}$$

The corresponding expressions of Ref. 1 have a missing term in the denominator of the formula for λ_{n1} and missing terms in both numerator and denominator of the formula for λ_{n2}.

The results of perturbation analyses, including terms up to the second order, for the first three frequencies in the case in which the unperturbed problem has parameters $\alpha = 1.0$, $\epsilon = 0$, $\beta = 0$ are shown in comparison with the exact solutions in Figs. 1-3 as a function of the perturbation parameter ϵ. Unlike the corresponding curves shown in Ref. 1, it can be seen that the perturbation theory gives reasonable approximation to the exact solutions for small values of ϵ, as it should. Moreover, the first and second derivatives of the curves of frequency parameter from perturbation theory apparently are equal to those obtained from the corresponding exact solutions at $\epsilon = 0$, as is required by the fundamental concept of perturbation analysis. In Ref. 1, the curves of λ vs ϵ for $\beta = 0$ obviously have not only the wrong slope but also second derivatives of the wrong sign.

Two factors may mask the errors in the perturbation theory numerical results for the cases in which $\beta \neq 0$, since in these cases Ref. 1 shows good agreement between the frequencies derived from second-order perturbation theory and the exact values. First, the larger numerical values of terms in β in the denominator of the formula for λ_1 and in both the numerator and denominator of the formula for λ_2 would tend to reduce the effects of the missing terms. Second, as is evident from the curves of frequency based on the exact solution in Ref. 1 for $\beta \neq 0$, the magnitudes of the changes in frequencies with ϵ are reduced substantially. Physically this corresponds to the well-known effect of large concentrated masses and rotational inertias in producing effective nodes in the normal modes. Addition of mass at a true translational node has no effect on the frequency to which that node belongs, and mass at an effective node (a point at which the vibration amplitude is small in comparison with its average value) produces small effects. Rotational inertia at node similarly has no effect when added at a true rotational node and small effect when added at an effective node. The combined effects of additions of mass and rotational inertia to the tip of a beam have been discussed in terms of a simplified model by Rayleigh (Ref. 2, pp. 289-291).

There are, in addition, other anomalies in the curves presented in Ref. 1 which cannot be explained in terms of the corrections to the formulas for λ_1 and λ_2 just given. For example, numerical or graphical errors must be the cause of the apparent variation in frequency parameter with ϵ for the higher modes with $\beta = 0.05$, $\alpha = 0$ in Fig. 3 of Ref. 1. When $\alpha = 0$, the tip mass is zero, and its displacement ϵ has zero effect according to both the exact theory and perturbation theory.

Refernces

[1] Bhat, R. and Kulkarni, M. A., "Natural Frequencies of a Cantilever with Slender Tip Mass," *AIAA Journal*, Vol. 14, April 1976, pp. 536-537.

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Comment on "Analysis of Transonic Cascade Flow Using Conformal Mapping and Relaxation Techniques"

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THE bilinear transformation of Ref. 1, described in detail by Eqs. (3) and (4a-e), can be given in a simpler form by using the properties of points symmetrical with respect to the unit circle:

$$\eta = \frac{(\zeta - B)C - (\zeta\bar{B} - 1)\,|C|S}{(\zeta\bar{B} - 1)\,|C| - (\zeta - B)CS}$$

where

$$C = \frac{A\bar{B} - 1}{A - B}$$

$$S = |C| - \sqrt{|C|^2 - 1}$$

In the original transformation of Ref. 1 the constants β and γ become singular if $B = -A$. Also, the constant S is unnecessarily defined as a minimum of $\sqrt{|\chi + (\chi^2 - 1)^{1/2}|}$ and $\sqrt{|\chi - (\chi^2 - 1)^{1/2}|}$ since the first expression is greater than one and the second less than one if $|A|, |B| < 1$.

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Errata

MHD Oscillatory Flow Past a Semi-Infinite Plate

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[AIAA J. **15**, 457-458 (1977)]

EQUATION (3) will read as follows

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{u}{C_p}\left[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{\sigma B_0^2}{\rho}U\right] + \frac{\sigma B_0^2}{\rho C_p}u^2 - \frac{1}{C_p}U\frac{\partial U}{\partial t} \quad (3)$$

The additional last term on the right-hand side of Eq. (3) does not affect Eqs. (6-11).

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